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The weak angle γ from one–particle inclusive CP asymmetries in the B_s^0 system.

Xavier Calmet
Ludwigs-Maximilians-Universität, Sektion Physik, Theresienstr. 37, D-80333 München, Germany

Abstract

We compute CP asymmetries of one-particle inclusive $B_s^0 \to D_s X$ decay rates. We find that the weak angle γ could in principle be extracted from these asymmetries although they are very small. Taking the decay width difference into account, we find that the time integrated CP asymmetries are of the order $\mathcal{A} = 1.4\ 10^{-4} \sin(\gamma)$. Some large uncertainties remain due to the lack of experimental data.

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1 Introduction

Some time ago it was proposed by Beneke, Buchalla and Dunietz [1] to study mixing induced CP violation in B mesons by using asymmetries of inclusive decay rates. This method although theoretically clean suffers from an experimental difficulty since it supposes a completely inclusive measurement. In order to solve this problem, we propose to study CP asymmetries of one-particle inclusive decays. In this paper, we shall consider CP asymmetries in the B_s^0 system. This can also be done in the B_d system [3]. We shall study time integrated and time dependent CP asymmetries of one-particle inclusive $B_s^0 \to D_s X$ decays. The technique involved is nevertheless theoretically not as clean as the one used in [1] for inclusive decays. Nevertheless, the measurement of CP asymmetries of one-particle inclusive decay widths would be clean.

We shall not neglect the decay width difference in the B_s^0 system, since it is sizable although recent calculations [8] show that the decay width difference in that system could be smaller than previously expected. A method [4] was recently proposed to calculate the decay rates appearing in these asymmetries. The predicted decay rates are compatible with current experimental knowledge and we can therefore be confident that the results we obtain for the CP asymmetries will be meaningful.

We shall concentrate on asymmetries involving one-particle inclusive decays of a B_s^0 meson. We start by introducing our notations, in section 3 we shall develop the formalism needed for time dependent CP asymmetries in one-particle CP asymmetries. We then present and discuss our results and conclude.

2 Definitions

In this section we shall introduce our notations. We basically use notations similar to those introduced in [1]. The proper time evolution of an initial pure B_s^0 or $\bar{B}_s^0$ reads

$$\begin{aligned} \mid B^0_{s\ phys}(t)\rangle &= g_+(t) \mid B_s^0\rangle - \frac{q}{p} g_-(t) \mid \bar{B}_s^0\rangle \\ \mid \bar{B}^0_{s\ phys}(t)\rangle &= -\frac{p}{q} g_-(t) \mid B_s^0\rangle + g_+(t) \mid \bar{B}_s^0\rangle, \end{aligned} \tag{1}$$

where the time dependent functions

$$g_+(t) = e^{-iMt-\frac{1}{2}\Gamma t}\left[\cosh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta M t}{2} + i\sinh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta M t}{2}\right] \tag{2}$$

and

$$g_-(t) = e^{-iMt-\frac{1}{2}\Gamma t}\left[\sinh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta M t}{2} + i\cosh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta M t}{2}\right] \tag{3}$$

describe the particle anti-particle mixing. The mass difference ΔM and the width difference $\Delta\Gamma$ between the neutral B_s^0 mesons are given by

$$\begin{aligned} \Delta M &= M_H - M_L \\ \Delta\Gamma &= \Gamma_H - \Gamma_L. \end{aligned} \tag{4}$$

The off diagonal term of the mass-mixing matrix is given by $M_{12} + i\Gamma_{12}$. We define the quantity

$$\frac{q}{p} = \frac{\Delta M - i/2\Delta\Gamma}{2\,(M_{12} - i/2\Gamma_{12})} = \frac{M_{12}^*}{|M_{12}|}\left(1 - \frac{1}{2}a\right), \qquad a = \mathrm{Im}\frac{\Gamma_{12}}{M_{12}}. \tag{5}$$

The second expression for q/p in equation (5) is valid to first order in the small quantity $\Gamma_{12}/M_{12} = \mathcal{O}(m_b^2/m_t^2)$.

3 CP asymmetries in one-particle inclusive decays

In this section, we shall consider CP asymmetries of rates of one-particle inclusive $B_s^0 \to D_s X$ decays. The time dependent CP asymmetry is defined by

$$\mathcal{A}_{CP}(t) = \frac{\Gamma(B_s^0(t) \to D_s X) - \Gamma(\bar{B}_s^0(t) \to \bar{D}_s \bar{X})}{\Gamma(B_s^0(t) \to D_s X) + \Gamma(\bar{B}_s^0(t) \to \bar{D}_s \bar{X})}. \tag{6}$$

In the following, we shall neglect all effects due to direct CP violation. The time dependent decay widths therefore read

$$\begin{aligned} \Gamma(B_s^0(t) \to D_s X) = |g_+(t)|^2 \Gamma(B_s^0 \to D_s X) + |\frac{q}{p} g_-(t)|^2 \Gamma(B_s^0 \to \bar{D}_s X) \\ -2\mathrm{Re}\,\left(g_+^\star(t)\frac{q}{p} g_-(t) T_{D_s}^{B_s^0\ \bar{B}_s^0}\right) \end{aligned} \tag{7}$$

and

$$\begin{aligned} \Gamma(\bar{B}_s^0(t) \to \bar{D}_s \bar{X}) = |g_+(t)|^2 \Gamma(B_s^0 \to D_s X) + |\frac{p}{q} g_-(t)|^2 \Gamma(B_s^0 \to \bar{D}_s X) \\ -2\mathrm{Re}\,\left(g_+^\star(t)\frac{p}{q} g_-(t) T_{\bar{D}_s}^{\bar{B}_s^0\ B_s^0}\right). \end{aligned} \tag{8}$$

The decay widths appearing in these formulas were calculated in [4]. It remains to compute the transition matrix elements of $\Delta B_s^0 = 2$ given by

$$\begin{aligned} T_{D_s}^{B_s^0\ \bar{B}_s^0} = \frac{1}{2m_{B_s^0}} \int d^4x \int d\phi_{D_s} \sum_X (2\pi)^4 \delta^4(P_{B_s^0} - P_{D_s} - P_X) \\ \langle B_s^0 | H_{eff}(x) | D_s X\rangle \langle D_s X | H_{eff}^\dagger(0) | \bar{B}_s^0 \rangle \end{aligned} \tag{9}$$

and

$$T_{\bar{D}_s}^{\bar{B}_s^0\ B^0{}_s} = \frac{1}{2m_{B_s^0}}\int d^4x \int d\phi_{\bar{D}_s}\sum_{\bar{X}}(2\pi)^4\delta^4(P_{B_s^0} - P_{\bar{D}_s} - P_{\bar{X}}) \qquad (10)$$
$$\langle\bar{B}_s^0|H_{eff}(x)|\bar{D}_s\bar{X}\rangle\langle\bar{D}_s\bar{X}|H_{eff}^\dagger(0)|B_s^0\rangle,$$

where $d\phi_{D_s}$ is the phase space of the D_s meson. The part of the effective Hamiltonian which is relevant for this work is given by

$$\begin{aligned} H_{eff} = &\frac{G_F}{\sqrt{2}}V_{cb}^*V_{cs}\left(C_1(\bar{b}c)_{V-A}(\bar{c}s)_{V-A} + C_2(\bar{b}T^a c)_{V-A}(\bar{c}T^a s)_{V-A}\right) \qquad (11)\\ &+\frac{G_F}{\sqrt{2}}V_{cb}^*V_{us}\left(C_1(\bar{b}c)_{V-A}(\bar{u}s)_{V-A} + C_2(\bar{b}T^a c)_{V-A}(\bar{u}T^a s)_{V-A}\right)\\ &+\frac{G_F}{\sqrt{2}}V_{ub}^*V_{cs}\left(C_1(\bar{b}u)_{V-A}(\bar{c}s)_{V-A} + C_2(\bar{b}T^a u)_{V-A}(\bar{c}T^a s)_{V-A}\right)\end{aligned}$$

where $(\bar{q}_1q_2)_{V-A}$ stands for $(\bar{q}_1\gamma^\mu(1-\gamma_5)q_2)$, the T^a matrices are the $SU(3)_C$ Gell-Mann matrices and C_1 and C_2 are the Wilson coefficients. The matrix elements $T_{D_s}^{B_s^0\ \bar{B}_s^0}$ and $T_{\bar{D}_s}^{\bar{B}_s^0\ B^0{}_s}$ can be parametrized in the same way as the wrong charm decay in [2]. Applying the Fierz transformation we can rewrite the operators into the following form $(\bar{b}s)_{V-A}(\bar{c}c)_{V-A}$, $(\bar{b}s)_{V-A}(\bar{u}c)_{V-A}$ and $(\bar{b}s)_{V-A}(\bar{c}u)_{V-A}$. Using the large N_C limit, we can factorize $T_{\bar{D}_s}^{\bar{B}_s^0\ B_s^0}$

$$T_{\bar{D}_s}^{B_s^0\ \bar{B}_s^0}(M^2) = \sum_q \frac{1}{2m_{B_s^0}}\frac{G_F^2}{2}|C_1|^2 V_{cb}^*V_{qs}V_{qb}^*V_{cs} \qquad (12)$$
$$\int d^4Q\, K^{\mu\nu}(p_{B_s^0}, M, Q)\int d\phi_{D_s}P_{\mu\nu}^{(q)}(Q)$$

with

$$K_{\mu\nu}(p_{B_s^0}, M, Q) = \sum_X (2\pi)^4\delta^4(p_{B_s^0} - p_X - Q) \qquad (13)$$
$$\langle\bar{B}_s^0(p_{B_s^0})|(\bar{b}\gamma_\mu(1-\gamma_5)s)|X\rangle\langle X|(\bar{b}\gamma_\nu(1-\gamma_5)s)|B_s^0(p_{B_s^0})\rangle$$

and

$$P_{\mu\nu}^{(q)}(Q) = \sum_{X'}(2\pi)^4\delta^4(Q - p_{D_s} - p_{X'}) \qquad (14)$$
$$\langle 0|(\bar{c}\gamma_\mu(1-\gamma_5)q)|\bar{D}_s(p_{\bar{D}_s})X'\rangle\langle\bar{D}_s(p_{\bar{D}_s})X'|(\bar{q}\gamma_\nu(1-\gamma_5)c)|0\rangle.$$

We get a similar expression for $T_{D_s}^{B_s^0\ \bar{B}_s^0}(M^2)$. The nature of the quark q depends on the operator under consideration. The tensor $K_{\mu\nu}(p_{B_s^0}, M, Q)$ is fully inclusive and can be parametrized using the decay constant $f_{B_s^0}$ of the B_s^0 meson and $P_{\mu\nu}^{(q)}(Q)$ is very similar to the expression we had encountered in the calculation of the wrong charm decay width [2]. It involves a projection on a state containing a

D_s meson. A priori we do not know how to calculate that kind of matrix element, a solution is to contract the spinor indices as in the parton calculation and to multiply this tensor by a channel dependent form factor f_q which has to be fitted. We obtain

$$P^{(q)}_{\mu\nu}(p_D,Q) = 2\pi\,\delta\Big((Q-p_D)^2 - m_q^2\Big)\,\mathrm{Tr}\Big\{\not{p}_D\,\gamma_\mu\,(\not{Q}-\not{p}_D)\,\gamma_\nu\Big\} f_q. \tag{15}$$

We can reproduce the inclusive case by setting $f_q = 1$.

3.1 CP asymmetry $\Gamma(B_s^0(t)\to D_s^+X)$ vs. $\Gamma(\bar{B}_s^0(t)\to D_s^-\bar{X})$

The CP asymmetry reads

$$\mathcal{A}(t) = \frac{\Gamma(B_s^0(t)\to D_s^+X) - \Gamma(\bar{B}_s^0(t)\to D_s^-\bar{X})}{\Gamma(B_s^0(t)\to D_s^+X) + \Gamma(\bar{B}_s^0(t)\to D_s^-\bar{X})}. \tag{16}$$

We have two quark transitions contributing to $T^{B_s^0\ \bar{B}_s^0}_{D_s^+}$, namely $b\to c\bar{c}s$ interfering with itself and $b\to u\bar{c}s$ interfering with $b\to c\bar{u}s$. These matrix elements can be modeled in the same way as it was proposed in [4]. It corresponds to a rescaling of the parton calculation. The complication due to isospin symmetry which led to the introduction of strong phases in [3] is not present in the B_s^0 system. We can use the same parameterization for the $\Delta B_s^0 = 2$ as the one we have used for the wrong charm decay widths [4]. The parton calculation was performed in [1]. We obtain

$$\begin{aligned} T^{B_s^0\ \bar{B}_s^0}_{D_s^+} = {} & -\frac{G_F^2 m_b^2}{24\pi M_{B_s^0}} f^2_{B_s^0} M^2_{B_s^0} V_{cb}V^*_{us}V_{ub}V^*_{cs}(1-z)^2 \\ & \cdot\left(C_1^2(1-z)B + C_1^2(1+2z)\frac{M^2_{B_s^0}}{m_b^2}2B^0_S\right)\mathcal{G} \\ & -\frac{G_F^2 m_b^2 f^2_{B_s^0}}{24\pi M_{B_s^0}} M^2_{B_s^0}(V^*_{cs}V_{cb})^2\sqrt{1-4z} \\ & \cdot\left[(1-4z)C_1^2 B + (1+2z)C_1^2\frac{M^2_{B_s^0}}{m_b^2}2B_S\right]\mathcal{F}, \end{aligned} \tag{17}$$

in the leading order of the short distance expansion and where $\mathcal{F}$ and $\mathcal{G}$ are decay channel dependent non-perturbative form factors. We shall define $\mathcal{F}$ and $\mathcal{G}$ later. The parameters B and B_S are the bag factors and z is defined by

$$z = \frac{m_c^2}{m_b^2}. \tag{18}$$

We have neglected the penguin operators. We also have two operators contributing to $T^{\bar{B}_s^0\ B^0{}_s}_{D_s^-}$, as in the previous case $b\to c\bar{c}s$ interfering with itself and $b\to c\bar{u}s$

interfering with $b \to u\bar{c}s$. In the leading order of the short distance expansion we then have

$$T_{D_s^-}^{\bar{B}_s^0\ B_s^0} = -\frac{G_F^2 m_b^2}{24\pi M_{B_s^0}} f_{B_s^0}^2 M_{B_s^0}^2 V_{cb}^* V_{us} V_{ub}^* V_{cs} (1-z)^2 \tag{19}$$
$$\cdot\left(C_1^2(1-z)B + C_1^2(1+2z)\frac{M_{B_s^0}^2}{m_b^2} 2B_S\right)\mathcal{G}$$
$$-\frac{G_F^2 m_b^2 f_{B_s^0}^2}{24\pi M_{B_s^0}} M_{B_s^0}^2 (V_{cb}^* V_{cs})^2 \sqrt{1-4z}$$
$$\cdot\left[(1-4z)C_1^2 B + (1+2z)C_1^2 \frac{M_{B_s^0}^2}{m_b^2} 2B_S\right]\mathcal{F},$$

where $\mathcal{F}$ and $\mathcal{G}$ are decay channel dependent non-perturbative form factors. The function $\mathcal{F}$ was given in [4]. We had

$$\mathcal{F}^{B_s^0 D_s^+} = \mathcal{F}^{B_s^0 D_s^-} = 4f, \tag{20}$$

where $f = 0.121$ was fitted in [2]. The value of this parameter could be slightly different in the B_s^0 case and will eventually have to be extracted from experiment. It can be extracted easily from the decay channel $B_s^0 \to D_s^+ X$. For the decays involving a D_s^* we have

$$\mathcal{F}^{B_s^0 D_s^{*+}} = \mathcal{F}^{B_s^0 D_s^{*-}} = 3f. \tag{21}$$

We shall make the very same assumptions to model $\mathcal{G}$ as we made to model $\mathcal{F}$. To do so we have to consider the contribution of $b \to u\bar{c}s$ to the decay rates of the form $\Gamma(B_s^0 \to D_s X)$ which we shall denote by $\Gamma(B_s^0 \to D_s X)^{b\to u\bar{c}s}$. Using the same model as the one we used for the wrong charm case in [2], we have

$$\Gamma(B_s^0 \to D_s^{*+} X)^{b\to u\bar{c}s} = 3\Gamma(B_s^0 \to D_s^+ X)_{dir}^{b\to u\bar{c}s}, \tag{22}$$

where $\Gamma(B_s^0 \to D_s^+ X)_{dir}^{b\to u\bar{c}s} = \Gamma_{dir}^{b\to u\bar{c}s}$ is the direct contribution to this decay over the current $b \to u\bar{c}s$. If we take into account the contribution from the decay over a D_s^{*+}, we obtain

$$\Gamma(B_s^0 \to D_s^+ X)^{b\to u\bar{c}s} = 4\Gamma_{dir}^{b\to u\bar{c}s} \tag{23}$$

since a D_s^{*+} always decays into a D_s^+. We assume that the direct contribution $\Gamma_{dir}^{b\to u\bar{c}s}$ to $\Gamma(B_s^0 \to D_s^+ X)^{b\to u\bar{c}s}$ is equal to the direct contribution to $\Gamma(B^0 \to D^+ X)^{b\to u\bar{c}s}$ which is certainly the case in the limit of the heavy quark symmetry. We assume that every c quark eventually hadronizes into a D meson. Spin counting and isospin symmetry in the decays $B \to D^* X$ and $B \to D_{dir} X$ through the transition $b \to u\bar{c}s$ allow us to deduce easily that Γ_{dir} is equal to 1/8 times the

result of the parton calculation. We then obtain the following non-perturbative form factors

$$\mathcal{G}^{B_s^0 D_s^+} = \mathcal{G}^{B_s^0 D_s^-} = 1/2 \tag{24}$$

and

$$\mathcal{G}^{B_s^0 D_s^{*+}} = \mathcal{G}^{B_s^0 D_s^{*-}} = 3/8. \tag{25}$$

In the B_s^0 system, we can use the following approximations

$$a = 0 \quad \text{and} \quad \frac{M_{12}^*}{|M_{12}|} = \frac{M_{12}}{|M_{12}|} = 1. \tag{26}$$

These are very good approximations, theory predicts $a < 10^{-3}$, we then neglect the weak phase in $\frac{M_{12}^*}{|M_{12}|}$. We see that the only weak phases appearing are $e^{i\gamma}$ in equation (19) and $e^{-i\gamma}$ in equation (17). We can therefore rewrite $T_{D_s}^{B_s^0\ \bar{B}_s^0}$ and $T_{\bar{D}_s}^{\bar{B}^0\ B^0}$ as

$$T_{D_s^+}^{B_s^0\ \bar{B}_s^0} = n_1 + e^{-i\gamma} n_2 \tag{27}$$

and

$$T_{D_s^-}^{\bar{B}_s^0\ B_s^0} = n_1 + e^{i\gamma} n_2, \tag{28}$$

where n_1 corresponds to the contribution from $b \to c\bar{c}s$ interfering with it-self and $e^{i\gamma} n_2$ corresponds to the contribution from $b \to c\bar{u}s$ interfering with $b \to u\bar{c}s$. Inserting the formulas for the time dependent decay rates in the definition for the CP asymmetry, we obtain

$$\mathcal{A}_{CP}(t) = \frac{-2n_2 \sin(\Delta M t)\sin(\gamma)}{M_1(t) 2\Gamma(B_s^0 \to D_s^+ X) + M_2(t) 2\Gamma(B_s^0 \to D_s^- X) + M_3(t)}, \tag{29}$$

where $M_1(t), M_2(t)$ and $M_3(t)$ are given by

$$M_1(t) = \cos^2\left(\frac{\Delta M t}{2}\right) + \sinh^2\left(\frac{\Delta\Gamma t}{4}\right), \tag{30}$$

$$M_2(t) = \sin^2\left(\frac{\Delta M t}{2}\right) + \sinh^2\left(\frac{\Delta\Gamma t}{4}\right) \tag{31}$$

and

$$M_3(t) = -(2n_1 + 2n_2)\sinh\left(\frac{\Delta\Gamma t}{2}\right). \tag{32}$$

The time integrated CP asymmetry is the given by

$$\mathcal{A}_{CP} = \frac{-2n_2 \frac{x}{1+x^2}\sin(\gamma)}{N_1(x,y)\Gamma(B_s^0 \to D_s^+ X) + N_2(x,y)\Gamma(B_s^0 \to D_s^- X) + N_3(x,y)}, \tag{33}$$

where $N_1(x,y)$, $N_2(x,y)$ and $N_3(x,y)$ are given by

$$N_1(x,y) = \frac{2+x^2}{1+x^2} + \frac{y^2}{4-y^2}, \tag{34}$$

$$N_2(x,y) = \frac{x^2}{(1+x^2)} + \frac{y^2}{(4-y^2)} \tag{35}$$

and

$$N_3(x,y) = -\frac{2y}{4-y^2}\left(2n_1 + 2n_2\cos(\gamma)\right), \tag{36}$$

where we have introduced the parameters $x = \Delta M/\Gamma$ and $y = \Delta\Gamma/\Gamma$ which can be measured. At the present time, there is only a lower bound for the parameter x in the B_s^0 system namely $|x| > 14$ [6]. For numerical calculations, we shall use $|x| = 20$ [7]. It is also not clear what the actual value of y is, for numerical calculations, we shall use the value computed in [8], $|y| = 0.054$. We set $B_S = B = 1$, $m_b = 4.8$ GeV, $m_{B_s^0} = 5.3693$ GeV , $m_c = 1.4$ GeV, $m_{D_s} = 1.9685$ GeV, $f_{B_s^0} = 210$ MeV and $C_1 = 1$. We obtain numerically

$$\begin{aligned} n_1 &= -4.931\ \%, \\ n_2 &= -9.526\ 10^{-2}\ \%, \end{aligned} \tag{37}$$

where n_1 and n_2 are normalized to the decay width of the B_s^0 meson, using $\tau_{B_s^0} = 1.54\ 10^{-12}$ s. The decay width needed were computed in [4], we had

$$\begin{aligned} \Gamma(B_s^0 \to D_s^+ X) &= 3.3\% \\ \Gamma(B_s^0 \to D_s^- X) &= 64.9\%. \end{aligned} \tag{38}$$

The time integrated CP asymmetry is then given by

$$\mathcal{A}_{CP} = 1.4\ 10^{-4}\sin(\gamma), \tag{39}$$

neglecting the term proportional to $\cos(\gamma)$ in the denominator.

3.2 CP asymmetry $\Gamma(B_s^0(t) \to D_s^- X)$ vs. $\Gamma(\bar{B}_s^0(t) \to D_s^+ \bar{X})$

In this case the CP asymmetry reads

$$\mathcal{A}(t) = \frac{\Gamma(B_s^0(t) \to D_s^- X) - \Gamma(\bar{B}_s^0(t) \to D_s^+ \bar{X})}{\Gamma(B_s^0(t) \to D_s^- X) + \Gamma(\bar{B}_s^0(t) \to D_s^+ \bar{X})}. \tag{40}$$

We need to parameterize $T_{D_s^-}^{B_s^0\ \bar{B}_s^0}$ and $T_{D_s^+}^{\bar{B}_s^0\ B_s^0}$. They are given by

$$
\begin{aligned}
T_{D_s^-}^{B_s^0\ \bar{B}_s^0} &= \left(T_{D_s^-}^{\bar{B}_s^0\ B_s^0}\right)^\dagger = n_1 + e^{-i\gamma} n_2 \\
T_{D_s^+}^{\bar{B}_s^0\ B_s^0} &= \left(T_{D_s^+}^{B_s^0\ \bar{B}_s^0}\right)^\dagger = n_1 + e^{i\gamma} n_2.
\end{aligned}
\tag{41}
$$

The form factors appearing in the transition matrix element were given previously. The time dependent CP asymmetry then reads

$$
\mathcal{A}_{CP}(t) = \frac{-2n_2 \sin(\Delta M t)\sin(\gamma)}{M_1(t)2\Gamma(B_s^0 \to D_s^- X) + M_2(t)2\Gamma(B_s^0 \to D_s^+ X) + M_3(t)}, \tag{42}
$$

where $M_1(t)$, $M_2(t)$ and $M_3(t)$ were defined in equations (30), (31) and (32). The time integrated CP asymmetry is then given by

$$
\mathcal{A}_{CP} = \frac{-2n_2 \frac{x}{1+x^2}\sin(\gamma)}{N_1(x,y)\Gamma(B_s^0 \to D_s^- X) + N_2(x,y)\Gamma(B_s^0 \to D_s^+ X) + N_3(x,y)}, \tag{43}
$$

where $N_1(x,y)$, $N_2(x,y)$ and $N_3(x,y)$ are given in equations (34), (35) and (36). The time integrated CP asymmetry is numerically given by

$$
\mathcal{A}_{CP} = 1.4\ 10^{-4}\sin(\gamma), \tag{44}
$$

neglecting the term proportional to $\cos(\gamma)$ in the denominator.

3.3 CP asymmetry $\Gamma(B_s^0(t) \to D_s^* X)$ vs. $\Gamma(\bar{B}_s^0(t) \to \bar{D}_s^* \bar{X})$

We can without any difficulty deduce from the preceding sections the formulas for the CP asymmetries in the one-particle inclusive decays of the form $\Gamma(B_s^0(t) \to D_s^* X)$ versus $\Gamma(\bar{B}_s^0(t) \to \bar{D}_s^* \bar{X})$. We have

$$
\begin{aligned}
n_1 &= -3.698\ \% \\
n_2 &= -7.145\ 10^{-2}\ \%,
\end{aligned}
\tag{45}
$$

which are normalized to the decay width of the B_s^0 meson. The decay widths needed were computed in [4], we had

$$
\begin{aligned}
\Gamma(B_s^0 \to D_s^{*+} X) &= 2.5\ \% \\
\Gamma(B_s^0 \to D_s^{*-} X) &= 49.6\ \%.
\end{aligned}
\tag{46}
$$

The next CP asymmetry we shall consider is defined by

$$
\mathcal{A}(t) = \frac{\Gamma(B_s^0(t) \to D_s^{*-} X) - \Gamma(\bar{B}_s^0(t) \to D_s^{*+} \bar{X})}{\Gamma(B_s^0(t) \to D_s^{*-} X) + \Gamma(\bar{B}_s^0(t) \to D_s^{*+} \bar{X})}. \tag{47}
$$

The time integrated CP asymmetry is then given by

$$\mathcal{A}_{CP} = 1.4\ 10^{-4} \sin(\gamma), \tag{48}$$

neglecting the term proportional to $\cos(\gamma)$ in the denominator. For the asymmetry defined by

$$\mathcal{A}(t) = \frac{\Gamma(B_s^0(t) \to D_s^{*-}X) - \Gamma(\bar{B}_s^0(t) \to D_s^{*+}\bar{X})}{\Gamma(B_s^0(t) \to D_s^{*-}X) + \Gamma(\bar{B}_s^0(t) \to D_s^{*+}\bar{X})}, \tag{49}$$

we obtain

$$\mathcal{A}_{CP} = 1.4\ 10^{-4} \sin(\gamma), \tag{50}$$

neglecting the term proportional to $\cos(\gamma)$ in the denominator.

4 Discussion of the results

We see that in principle we can extract information on $\sin(\gamma)$ from the asymmetries calculated in the previous section. But they are small, nevertheless the decay widths involved are sizable. It has been proposed to extract information on $\sin(\gamma)$ from exclusive decays (see e.g. [9]) but the decay widths involved are very small, typically of the order 10^{-4} and one would have to deal with strong phases which would make the extraction of $\sin(\gamma)$ even more difficult. It could therefore be worth to try to extract $\sin(\gamma)$ from one-particle inclusive decays in the B_s^0 system. If the present method is chosen to extract $\sin(\gamma)$, it would be interesting to test its precision. This could be done by comparing the results obtained for $\sin(2\beta)$ in one-particle inclusive CP asymmetries in the B_d system [3] with some more conventional extraction technique like the "gold-plated" $B \to J/\Psi K_S$, although one-particle inclusive CP asymmetries in the B_d system are theoretically not as clean as the ones in the B_s^0 system due to the presence of strong phases [3].

We still have some large uncertainties, some of them due to the method. The corrections to the decay widths could be fairly large, in the worth case of the order of 30%. But remember that the decay widths calculated in [4] are compatible with current experimental knowledge. On the other hand, we have large experimental uncertainties in the values of x, y and $f_{B_s^0}$.

Time dependent CP asymmetries could also allow to extract $\sin(\gamma)$, but it is not yet clear if the oscillations can be resolved. Predictions depend on the decay width difference in the B_s^0 system and it has not yet been possible to measure this quantity.

A way to improve the magnitude of the CP asymmetries would be to do an anti-lepton tagging. The decay widths in the denominator are partially responsible for the low magnitude of the asymmetries, this is particularly true for the

CP asymmetries in the B_d^0 system considered in [3]. Thus, doing an anti-lepton tagging, would slightly improve their magnitude. We would then consider asymmetries of the type

$$\mathcal{A}(t) = \frac{\Gamma(B_s^0(t) \to D_s X)_{\mathrm{NL}} - \Gamma(\bar{B}_s^0(t) \to \bar{D}_s \bar{X})_{\mathrm{NL}}}{\Gamma(B_s^0(t) \to D_s X)_{\mathrm{NL}} + \Gamma(\bar{B}_s^0(t) \to \bar{D}_s \bar{X})_{\mathrm{NL}}}, \tag{51}$$

where NL stands for non-leptonic. In the B_s^0 system, we would have time integrated CP asymmetries of the order $2\ 10^{-4} \sin(\gamma)$. The factor $x = 20$ is responsible for the low magnitude of the asymmetries. In the B_d system, this effect would obviously be larger.

5 Conclusion

We have discussed CP asymmetries in one-particle inclusive $B_s^0 \to D_s X$ decays. The asymmetries are small but would allow to extract $\sin(\gamma)$ which is known to be difficult. So any new method is probably welcome. It has the advantage, in comparison to CP asymmetries of exclusive decays, to have some large decay widths and in comparison to CP asymmetries of inclusive decays, of being experimentally clean.

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